Modeling the Motion of Solid Objects Immersed in Viscoelastic Fluids

by

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Flows of viscoelastic fluids around immersed objects have long been a subject of interest in both experimental and theoretical studies. Such flows are used as benchmarking problems for validation of advanced numerical methods and the predictions of equations of state, as model systems for larger industrial processes, and as coarse-grained models for the movement of microorganisms and other active micro-scale objects (often referred to as "microswimmers") through biofluids and other viscoelastic media. Additionally, a better understanding of such flows offers the potential to improve the ways in which microrheology measurements are made and interpreted. This thesis aims to develop a flexible, adaptable framework for describing the motion of geometrically simple objects in weakly elastic fluids by adapting well-established and robust analytical methods.

The first part of this thesis focuses on the describing the motion of single spheres immersed in weakly viscoelastic fluids moving in arbitrary, time-dependent one- and two-dimensional trajectories. We show how perturbation methods can be used in conjunction with the Lorentz reciprocal theorem to determine a general relationship between the particle trajectory and the force exerted on it by the surrounding fluid at low Weissenberg numbers, as well as how an inverse relationship can be determined and used to analyze the trajectory of a particle propelled by some external force. Potential applications for this solution methodology are explored in detail, including the development of a framework for analyzing data from active microrheology experiments in the weakly nonlinear regime and the design of different forcing protocols for externally directed spherical microswimmers.

The next part of this thesis focuses on modeling an independently propelled force- and torque-free microswimmer composed of two counterrotating spheres of differing radii. Swimmers of this type, while ineffective in Newtonain fluids, are capable of propelling themselves through nonlinear viscoelastic fluids due to the imbalance of normal stress differences induced in the fluid by the rotating asymmetric body. The propulsion speed of such swimmers in a variety of configurations is determined using perturbation methods and an application of the reciprocal theorem. The impact of varying both the details of the swimmer geometry and the rheological properties of the surrounding fluid on the propulsion speed are described in detail.

The final part of this thesis focuses on the modeling and analysis of a long, slender mechanical resonator in both Newtonian and weakly viscoelastic fluids for use as a continuously deployed rheometer in industrial settings. Much like the motions of spherical particles discussed in the first parts of this thesis, the vibration of an immersed thin cantilever beam is another canonical problem in fluid mechanics, and we again show how well-established methods for analyzing this system can be adapted to better capture the influence of viscoelasicity. Ultimately, we leverage a derived relationship between the free-end displacement of a cantilever beam undergoing some known, externally driven vibration and the hydrodynamic force exerted on it by the surrounding fluid to accurately determine rheological properties of several fluids using both optical and electric measurements of the free-end displacement of a real mechanical resonator at its resonance frequency.

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